Mathematical sociology

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abstract The history, presuppositions, current developments, challenges and prospects for mathematical sociology are described. Common research threads in the field from its earliest post-Second World War days to the present are identified. A case for the importance of mathematical sociology to the future of sociological theory is made.

keywords mathematical sociology ◆ model building ◆ process ◆ sociological theory ◆ structure

Mathematical sociology weds mathematics and sociology to advance the scientific understanding of social structures and social processes. Within the broad arena of sociology, it stands in that corner defined by a generalizing orientation, by the belief that a science of social orders is possible, by a commitment to a logical derivation of empirical regularities from formally stated axioms or assumptions, and by a concern for the integration and unification of sociological theory. Mathematics in this context is used to directly formulate theory and derive testable hypotheses. Mathematics is also widely used in data analysis in the social sciences but such use is not intended to be in the service of theory formulation. Nevertheless, practitioners of quantitative data analysis and practitioners of mathematical sociology can overlap substantially since the requisite skill set, facility with mathematical formulations and the training to reason logically from premises, is common to both enterprises. This article surveys the field of mathematical sociology, its history, its presuppositions and current areas of development. It then outlines some of the basic challenges faced by mathematical sociology, closing with a commentary on its future directions.

As an intended scientific discipline, sociology is relatively new and mathematical sociology even newer. The canonical problems of the discipline are not agreed upon and a variety of theoretical perspectives exist as competing paradigms. The array of such general perspectives in sociology is what is usually meant by 'sociological theory'. At the level of macrosocial analysis, one perspective (functionalism) treats values in terms of their social integrative significance while another (conflict theory) treats values as weapons in struggles among groups. At the level of microsocial analysis, one perspective (exchange theory) treats social interaction in terms of interchange of resources of any kind, while another perspective (symbolic interactionism) treats social interaction as communication of social meanings. These perspectives and others have arisen in pursuit of the goal to formulate fundamental *sociological problems* – what sociology should explain and how. In turn, the use of mathematics in sociological theory has arisen in the context of attempting to find more effective means of the pursuit of explanatory goals, drawing upon but not limited to existing theoretical perspectives.

History

Interest in mathematical sociology took off in the post-Second World War period. The pre-war work of Rashevsky (1939a, 1939b, 1940a, 1940b, 1941, 1942) was emblematic of early efforts in mathematical sociology – problems were approached in a 'grand theory' fashion with little attention to relevant data. The post-war period, the 1950s and 1960s, was a

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fruitful one for mathematical sociology in part because theorists began by paying attention to significant empirical regularities, the precision of which virtually demanded formal analysis. Many of these regularities derived from laboratory studies of small group processes. Specifically, there were the studies of participation in task groups by Bales and colleagues (Bales, 1950; Bales et al., 1951), the studies of the effect of the structure of a communication network on group performance and individual outcome by Bavelas and his colleagues (Bavelas, 1948, 1950), Leavitt (1949, 1951), Smith (1950) and Bavelas and Barrett (1951) and the studies of conformity by Asch (1951, 1952, 1956).

The importance of small group research was clearly recognized in the title of one of the early books to organize mathematical thinking in sociology, *Types of Formalization in Small Group Research* (Berger et al., 1962). Useful to this day is their categorization of formal models by three types of primary goal: explication, representation and theoretical construct. In the first type mathematics is used in the 'explication, or rendering of precise meaning, to one or more basic concepts' (Berger et al., 1962: 7). Their illustration is the use of graph theory by Cartwright and Harary (1956) to render more precise the idea of balance, the primary driver in Heider's theory of interpersonal relations (1944, 1946, 1958). In the second type, 'the theorist attempts to represent in as precise and formally simple a manner as possible a recurrent but specific instance of an observed social phenomenon' (Berger et al., 1962: 7). The paradigmatic illustration is Cohen's (1958, 1963) Markov chain model of the Asch conformity process. In the third type, the theorist aims 'to provide a direct means of developing a *general* explanatory theory which formally accounts for a variety of observed processes' (Berger et al., 1962: 67; italics in original). The exemplar they discuss is the 'stimulus sampling' learning model of Estes and Burke (1955), a model proposed to explain outcomes in one type of learning experiment (discrimination learning) whose conceptual machinery was sufficiently general that it was elaborated to apply to multi-person situations (Atkinson and Suppes, 1958).

Another common route to mathematical models was to recast verbally stated theoretical claims in formal terms. Simon's (1952) formalization of Homans' (1950) theory is the classic example. The conceptual foundation of Homans' theory is a social system model in which there are two subsystems, external and internal. Each subsystem is described by three variables: activity, interaction and sentiment. In the external system, the variables are specified as task-related and consisting of interactions needed for the task; in the internal system, the three variables characterize interpersonal relations such as friendships. The general idea is that the internal relationships emerge out of the social activity and interaction required for adapting to the group's environment but then feed back to alter or reinforce that adaptation. For instance, a work group's strong internal ties may enable it to resist some new practice initiated by higher management (environment).

Homans made several claims about how *in general* activity is related to sentiment, sentiment to interaction, and so on from his analysis of five groups that had been subjects of detailed field studies by social scientists. Simon formalized these assertions by specifying mathematically the functional relationships between the variables of activity, sentiment and interaction and connecting them via differential equations into a dynamic system characterizing the group. The system of equations was analyzed for the conditions under which equilibrium points existed and were stable. The hope was that such conditions could then be interpreted in meaningful sociological terms that expanded the theoretical understanding of the establishment and maintenance of social groups.

Other foundational work appeared not in sociology journals but in the *Bulletin of Mathematical Biophysics*. In a series of papers, Anatol Rapoport and colleagues set out and explored formal models for potentially extremely large groups (Landau, 1952; Rapoport, 1951a, 1951b, 1953a, 1953b, 1953c, 1957, 1958, 1963; Rapoport and Horvath, 1961; Rapoport and Solomonoff, 1951). In particular Rapoport's random and biased net theory focused on modeling social networks – the structure of social ties of a particular type in a bounded population. The models used probabilistic formalisms first to describe properties of a random net and then to model departures from randomness by biases that made certain connections more likely than chance when the relational context was favorable. For instance, the idea that a person, ego, is more likely to chose another as a friend if the other has also chosen ego as a friend was formally expressed by a 'reciprocity' bias. The basic problem concerning Rapoport was the 'tracing' problem: how biases and the random chance of connection affected reachability, the proportion of nodes in a population that could be reached from a randomly selected starter node. The average proportions of newly reached nodes in each generation, as links were traced out from an arbitrary starting set, were termed the biased net 'structure statistics' (Fararo and Sunshine, 1964). Biases were defined in the context of the tracing procedure. Rapoport derived formal expressions for the structure statistics but only under some strong approximation assumptions that later research showed needed correction to improve fit to available data (Skvoretz, 1990).

By the 1960s and 1970s, a sufficient number of exemplars of the use of mathematics in sociology had accumulated that there was a spate of textbooks directed at introducing a reader to mathematical sociology: Coleman's *Introduction to Mathematical Sociology* (1964), Doreian's *Mathematics and the Study of Social Relations* (1970), Fararo's *Mathematical Sociology: An Introduction to Fundamentals* (1973) and *Mathematical Sociology* (1975) by Leik and Meeker, although the first text was arguably Karlsson's *Social Mechanisms: Studies in Sociological Theory* (1958). Second, there were several texts that either aimed for a wider audience of social scientists, such as *Mathematical Models in the Social Sciences* (1962) by Kemeny and Snell and *Introduction to Models in the Social Sciences* (1975) by Lave and March, or developed applications of a particular mathematical formalism for a more narrowly defined set of applications, such as Bartholomew's *Stochastic Models for Social Processes* (1967). Finally, mention should be made of Boudon's work using mathematics to explore aspects of mobility and educational stratification systems in *Mathematical Structures of Social Mobility* (1973) and *Education, Opportunity, and Social Inequality* (1974), a study which also employed numerical simulation.

Of the mathematical textbooks, the ones by Coleman and by Fararo were the most ambitious and comprehensive yet they were quite different efforts. Coleman's introduction grew out of immersion in large-scale survey analysis and a recognition that much of the data in sociology came in the form of proportions or percentages. Because of his early training as an engineer, Coleman was well aware that systematic measurement was necessary to scientific advance and that it was lacking in the social sciences – witness the title of his Chapter 2, 'The problems of quantitative measurement in sociology'. Measures based on counting, however, bypassed some of the problems facing social scientists trying to measure cherished concepts like solidarity or the division of labor. In Coleman's version of mathematical sociology, mathematical models of social phenomena were driven by this recognition that counting things in classes provided the systematic measurement basis necessary for the science of sociology. Hence his fundamental type of model was a continuous time, discrete state stochastic process in which persons' random movement from one state to another was determined probabilistically by endogenous and exogenous forces representing social effects like influence. Coleman's introduction did not aim to acquaint the reader to the variety of mathematics that could be used to formalize sociological theories.

Fararo was equally aware of the importance of systematic measurement to scientific advance yet the material he presented was much less driven by the measurement basis of counting and more oriented to a comprehensive survey of useful mathematical techniques in social science. Topics covered included stochastic process models, specifically Markov chains as source models for the formation of images of stratification, the formation of expectation states and the representation of social mobility; abstract algebra as source models for social relational structures like kinship; and the theory of games as proposed by Von Neumann and Morgenstern (1947) as source models for interactive choice situations or, more narrowly, rule-governed interactive choice situations, thought of as the core generator of sociological puzzles and problems. He also provided extensive background introduction to key mathematical concepts like mappings, sets, vectors, matrices, Markov chains and abstract groups. Fararo's introduction was squarely aimed at acquainting the reader with the variety of mathematics that could be used to formalize sociological theory and model sociological processes and structures.

The appearance of textbooks was one of the indicators marking the start of the institutionalization of mathematical sociology as a field within the discipline. Other indicators included the establishment of journals and professional associations and the founding of graduate programs. On the first point, the first issue of *The Journal of Mathematical Sociology* was published in 1971. Later in the decade, came the journal *Social Networks*, founded in 1978, followed by the annual publication *Advances in Group Processes*, founded in 1984, the journal *Rationality and Society*, founded in 1989, and then much later in 1998 the online *Journal of Artificial Societies and Social Simulation* and in 2000 the electronic journal of the International Network for Social Network Analysis, *Journal of Social Structure*. The formation of professional associations lagged about a decade behind these developments. Early on the scene was the Japanese Association for Mathematical Sociology founded in 1986. But it was not until 1997 that a Section on Mathematical Sociology of the American Sociological Association was established. In 2000, the section collaborated with the Japanese association to organize the first joint Japanese–American Conference on Mathematical Sociology (held in Hawaii). The activity of these groups is helping to change the earlier situation of the field, described by Fararo (1997) as one of lacking common identity and solidarity.

As early as 1970 the University of Pittsburgh had a mathematical sociology track in its PhD program largely due to the efforts of Tom Fararo, Pat Doreian, and somewhat later Norm Hummon. The most successful program of the day was associated with Harrison White at Harvard. Although White never authored a textbook in mathematical sociology, he provided key monograph-length exemplars of the creative use of mathematical concepts to model social structure and social process. Trained initially with a PhD in physics before taking a PhD in sociology, White's first monograph in mathematical sociology, *An Anatomy of Kinship* (1963), tackled a structure problem using abstract algebra to analyze kinship structures as compositions of elementary kinship relations and argued that different kinds of kinship structure emerge from placing different compositions of relations in equivalence to one another. A second monograph in 1970, *Chains of Opportunity*, tackled the process problem of social mobility in a well-defined career system and stood the prevailing conceptualization on its head. The typical approach viewed the flow of persons through a set of positions as the modeling problem, whereas White viewed it as the flow of a vacancy through that set of positions. He showed that the vacancy flow count could be modeled as a Markov process while creating by implication a flow of persons through positions that were non-Markovian and far more difficult to model. The program at Harvard under White's intellectual leadership produced an accomplished generation of mathematically inclined scholars: Phil Bonacich, Ron Breiger, Ivan Chase, Bonnie Erikson, Mark Granovetter, Joel Levine and Barry Wellman, among others.

Finally, mention should be made of two longstanding research programs that have been quite supportive of formal models in sociology, even if they would find it awkward to be claimed as research programs in mathematical sociology. The first and most prolific is the expectation states program based at Stanford University and founded by Joseph Berger, Bernard P Cohen and Morris Zelditch Jr (Correll and Ridgeway, 2003). The number of outstanding scholars trained by this program and associated with it is truly remarkable. The program's consistent focus has been on the development of power and prestige orders in task groups and the myriad ways in which exogenous status considerations and endogenous status processes impact an actor's position in the power and prestige order. The second program is that of David Heise and colleagues on affect control theory, a theory designed to model action and emotion based on the affect profiles of situational identities and activities (Robinson et al., 2006). Rooted in the classical tradition of symbolic interactionism, this research program uses control theory and its associated mathematical machinery of matrix algebra to understand how the meanings of separate elements comprising a situation (as measured by the semantic differential) combine to reinforce those fundamental sentiments. These two and a number of other theoretical research programs that employ mathematical model building are included in the edited volume *New Directions in Contemporary Sociological Theory* (Berger and Zelditch, 2002).

Overview and presuppositions

At the heart of the scientific research enterprise is the coordination of theory with relevant data. Scientists systematically collect information in order to test theories – sets of ideas about how the natural world works. Social groups and their workings are part of the natural world and the subject matter of the social sciences, sociology in particular. Verbal formulations of typical sociological theories are useful up to a point. However, precise understanding requires (among other things) that hypotheses be stated as precisely as possible and hence the utility of mathematics to sociology (and to all other sciences as well). Mathematics is particularly valuable to the social sciences where theories stated in ordinary language can easily import layers of hidden meaning and cultural content that skew testing and analysis. This claim is, of course, not widely shared by theorists who eschew formal argument.

This claim should also be distinguished from the point that mathematics brings value to social science via statistical techniques for data analysis. The difference between mathematical models and statistical models has been addressed in different ways by Collins (1988: Appendix A), Skvoretz (1998) and Sørenson (2009). Collins suggests that behind any statistical analysis is a substantive theory in which operative causal mechanisms produce random distributions and that explicitly building on such a theory should be a priority for social science. Sørenson argues that statistical models are often used because sociologists do not have good substantive models based in theory and so default to the statistician's recommendations. Finally, Skvoretz demonstrates how consequential the choice between a default statistical model and a theoretically derived model can be for assessing whether a theory is generally disconfirmed on one hand or clearly supported on the other. All three underscore the point that mathematical sociology's value lies on the theoretical side of the scientific enterprise rather than on its methodological side.

Fararo (1973) provides a useful overview of the presuppositions and steps involved in any research project in mathematical sociology. Following Toulmin (1953), Fararo holds that theoretical science searches for novel and systematic ways of representing phenomena. The representations give exact form to the phenomenon of interest. So, for instance, the conformity process as studied by Asch (1951) is a process by which a person makes a sequence of choices, each of which either conforms to a group's judgment or not. The representation of this process by a Markov chain, as proposed by Cohen (1963), gives exact form to this process – there are a number of states (four) that a person may occupy, each state corresponds to a particular response with probability 1, and between one choice and the next, a person may move from one state to another, the probabilities of movement dependent on the exact parameters of the Markov process and interpretable in terms of the referent phenomenon of conformity. Fararo notes that one advantage of formal representation is that it makes certain questions meaningful (how does the time to absorption into the state of permanent conformity depend on parameters?) and rules out others (does failure to conform to the group judgment indicate exceptional independence of character and leadership potential?). Another advantage is that the formal representation, if found to be generally descriptively adequate, allows the theorist to raise the deep question of why it should be so, the question of explanatory adequacy.

A generic method of representation provides, in Fararo's (1973: Sect. 1.3) terms, a framework and he embeds model building, the key step in any research project in mathematical sociology, in the context of framework construction. The research project begins implicitly or explicitly with the framework in a current state and that state, along with interest in some empirical phenomena, leads to a scientific problem. For example, the general framework which stipulates that influence flows through a group represented as a social network of significant connections among persons, along with the interest in the process of adoption of some new technology, leads to the scientific question of how adoption is driven by contagion from connections and/or by homophily or attribute similarity among persons (Aral et al., 2009). Model building then attempts to 'solve' the scientific problem. The activity of building models has three steps: (a) model setup, (b) model analysis and (c) model application. A valuable research project may stop with the first two of these steps: Simon's formalization of Homans' *Human Group* is an example.

Model application itself, as Fararo specifies, has four substeps: (a) identification of abstractions, (b) estimation of parameters, (c) calculation of predictions and (d) evaluation of goodness of fit. Skvoretz (1981) provides an exemplar for the model building stage: the research question is the distribution of participation in task focused groups of any size and the model is set up as probabilistic process in which the strength of a person's inclination to participate relative to the strengths of all others in the group determines the probability of participation and strength is determined by the person's comparative status in the group. Analysis derives relationships between the status composition of the group and the chances that persons occupying particular statuses in the group participate. The application phase uses data from administrative conferences in a psychiatric hospital – the operative status dimension is identified, parameters calibrating status effects on participation are estimated, predicted distributions are derived and, finally, comparisons of predicted to observed distributions assess fit.

Fararo (1973: 6) makes the important observation that feedback loops connect the steps in the research process as the results of a model evaluation can feed back to the first two steps of the model building and that in turn can feed back to reformulate the scientific question. Over time such reformulations of the scientific question can change the state of the framework suggesting different or more refined methods of representation that evoke a new cycle of model building efforts. Again work on accounting for status effects in the distribution of participation in task groups can serve as an example: limitations in the data addressed in Skvoretz (1981) led to new data collection efforts to evaluate the model (Smith-Lovin et al., 1986) and the evaluation led, in turn, to more sophisticated models (Skvoretz, 1988) but also to concern with emergent status distinctions from behavioral cues and their integration with exogenous diffuse status effects on participation. The scientific question is reformulated and the framework shifts to a discrete state stochastic process in which emergent and activated status orders (and thus states of the stochastic process) are represented by an evolving network of precedence relations (Skvoretz and Fararo, 1996).

It is in the model building phase that the full menu of options offered by modern mathematics comes into play. Following Fararo (2001) there are, broadly speaking, two types of models: process models and structure models. Process models aim to capture the trajectory of a system as it moves over time through a state space. Key elements of a process model are the conceptualization of the time domain as discrete or continuous, the definition of the states as discrete or continuous, the conceptualization of the states as probability distributions or deterministic positions, the specification of the change-of-state rules and the conceptualization of parameters used by these rules as discrete or continuous. Simon's process model, for example, falls in the category of continuous time, continuous and deterministic state, with continuous parameters in change-of-state rules embodied by differential equations. Key questions for the analysis of such models are the existence and stability of equilibrium states and their dependence on the relative values of key parameters. These equilibrium states may be specific positions or states or probability distributions over a set of states or positions.

These basics of a dynamic process model are useful not only to elucidate specific content areas but also as a metaphor for the aims of general theoretical sociology. Fararo (1989: 109) uses these basic ideas to define the four fundamental problems of theoretical sociology: (1) the existence and forms of social structures (are there equilibrium states of social processes?); (2) the stability of social structures (does the system return to an equilibrium state if small shocks move it out of that state?); (3) the comparative statics of social structure (how do the equilibrium states of social processes depend on parameters of the processes?); and (4) social change conceptualized as movement from one equilibrium state to another (what higher level process can produce movement from one such state to another via parametric change?). The last problem provides a formal viewpoint on the classic micro–macro problem in sociological analysis. It adds the intriguing idea that the connection is made through a feedback loop from the state space of the micro process to its parameter space which constitutes the state space of the macro process.

Structure models, on the other hand, focus on a set of abstract objects, held to represent some social phenomenon of interest, and study ways of characterizing important properties of these abstract objects. The notion of balance in signed graphs is one of the clearest examples of a structural model. The type of abstract object is a collection of nodes with pairwise edges between them that have either positive or negative valence, meaning that they have weights that can be interpreted as positive or negative constants (usually 1 or –1). Interpretively, the collection could be a group of persons and the edges connecting pairs who like or dislike each other. A much cited result of the analysis of this structural model is Cartwright and Harary's (1956) proof of the 'structure theorem': that the nodes in every balanced signed graph may be partitioned into two subsets (one of which may be empty) such that positive edges join nodes in the same subset and negative lines join nodes in different subsets.

Signed graphs are part of a larger family of structure models in which structure is represented by a network. Three other families of structure models are ones in which structure is represented by distributions, by rule systems (grammars) and by games (Fararo, 2001). Distribution models focus on properties of the (static) distribution of persons into positional categories. The classic use of these models is found in the work of Blau (1977), whose concern was how properties of these distributions affected intergroup relations. Rule system models find use in representing institutions as systems of social roles (Fararo and Skvoretz, 1984, 1986). Game-theoretic models have been widely used to represent interactive choice situations such as public goods problems and emergence of cooperation in prisoner's dilemmas and are much more prevalent in the work of economists than sociologists (Macy and Skvoretz, 1998). Game-theoretic concepts also play a role in the rational choice framework associated with the later work of James Coleman (1990) and others. In Coleman's framework, the structure model is defined by actors' initial endowments of (a) interests over events or resources and (b) control over those events or resources. Under a utility maximization assumption, control over events of lesser interest is traded for control over events of greater interest until a competitive equilibrium is reached. Coleman uses this model to analyze power, trust, norms and constitutions.

This categorization into process models vs structure models should not be pushed too far. Fararo (2001) includes a category of models combining process and structure. Indeed, there are research projects in mathematical sociology that have elements of both: for instance, the models developed in Skvoretz and Fararo (1996) for participation in task groups are process models in which the states through which a task group moves are represented by a structure model, namely, networks of directed ties of precedence connecting persons. Also the rational choice framework can incorporate process elements (Coleman, 1990: 899–931; Fararo and Skvoretz, 1993). Nevertheless the distinction is useful for those just learning about mathematical sociology and wanting to learn more about its signature achievements.

Perhaps the most recent development in mathematical sociology that represents the biggest departure from its classical foundations is the use of agent-based models to investigate social phenomena. This modeling framework developed outside of mathematical sociology, although one of its earliest successes, Schelling's segregation model (1971), was published in the first issue of the *Journal of Mathematical Sociology*. The use of this framework is so distinctive that the research area has its own name, computational sociology, a term first widely used by Hummon and Fararo (1995).

Agent-based models focus on systems consisting of multiple agents and the emergence of system regularities from local interactions between agents. Agents have internal states and behavioral rules and the rules may be fixed or changeable through experience and interaction. Agents are boundedly rational; they have only limited information processing and computational capacity. Agents interact in an environment that provides resources for their actions. Typically, agents and/or the rules they use thrive or die based upon their success in obtaining resources.

The setup of an agent-based model requires that simulation be used to analyze its consequences. In such a model, there are typically many agents and probabilistic considerations figure in the determination of who interacts with whom and in the determination of the changes of agent state. Mathematical analysis of such a system for equilibrium solutions is not feasible. The only way to explore logical consequences is through simulations. In this field, the design of the agents and the rules under which they interact are the assumptions of the formal theory and simulation plays the role of deduction from that theory. In general, the aim is to derive regularities at the aggregate level from the interaction of agents following relatively simple rules at the micro level. Such regularities are 'emergent' relative to the lower-level rules of interaction and agent state change and thus, in principle, not predictable from these rules. Therefore, simulation is used to detect such emergent regularities.

An agent-based research project has three phases: model setup, model implementation and execution and inductive analysis of model output. In the first phase, decisions are made about how agents may interact and what rules govern their changes of state. In the next phase, the system of agents is encoded in a computer program and then various 'runs' of the program made. In the last phase, the output is then analyzed for regularities that can be reasonably attributed to the underlying assumptions about the behavior constraints on agents encoded in the program. Care must be taken so that substantive meaning is not attributed to regularities that are artifacts of implementation. The modeling exercise is convincing when the assumptions about behavior are clear and intuitively reasonable or based clearly on existing theory, the program implementation is transparent, a full range of initial conditions and values of basic parameters is explored, clear regularities emerge and variation in these regularities can be interpreted in terms of the model's original assumptions. There remain difficult issues in the coordination of such models with observational data. Occasionally, the simple derivation of results by simulation is treated as an empirical test but this is a mistake – at best, such derivations can demonstrate plausibility, but full-scale testing of the model requires the usual steps of parameter estimation and derivation of specific empirical consequences. The problem of empirically testing agent-based models has been addressed recently in a special issue of *Ecology and Society* edited and introduced by Janssen and Ostrom (2006). They suggest four approaches including laboratory experiments, role-playing games, case studies and derivation of stylized facts.

It is clear that agent-based models straddle the distinction between process and structure models. They also formally address a fundamental problem in theoretical sociology, the macro–micro problem: namely, how is it that the micro-behavior of actors aggregates to macro-regularities of the system they compose – as Raub et al. (2011: 2) state: 'establishing micro–macro links to explain social macro-level phenomena as a result of the behavior of individual actors is a core aim of model building in sociology'. Both observations, along with plentiful computing power, are reasons for the accelerated interest and use of the agent-based modeling strategy in mathematical sociology. There is, possibly, an even deeper reason: agent-based models comprise the only way to achieve a scientific understanding of situations when an agent interacts with a moderate number of others over structured networks of connections, situations where according to Miller and Page (2007: 221) 'our traditional analytic tools break down'. In their view the traditional analytic tools of mathematical social science produce tractable models when either very few (usually two) or very many (an infinite number) of agents are postulated.

Agent-based models often make excellent use of game-theoretic formulations of situations of interaction. Game theory, with its working hypothesis of utility maximization, had its natural first home in economics. Sociologists have had much trouble accepting the utility maximization hypothesis and so much of rational choice theory, founded in an economic perspective on social life, has been criticized, occasionally unfairly, for this reason. However, the notion of bounded rationality, as developed by Simon in his *Models of Man* (1957), is far more acceptable to sociologists. The problem, however, is that while there is one clear route to optimization, there are many ways to be boundedly rational. Hence developing formal models that rest on boundedly rational agents is a challenge for agent-based models. In these models interactions are modeled as repeated plays of games in the formal sense and the model must be explicit about the ways in which agent rationality is bounded. For instance, the agents in Skvoretz and Fararo's early (1995) model of the evolution of reciprocity recall only the last two encounters and formulate a plan of action based on the outcomes in these last two encounters. The action plan specifies which alternative to select in the next encounter and the choice when coupled with the selection of the agent encountered determines payoffs to both parties according to the game matrix. Effectively in this more sociological approach, systems of agents 'solve' the game through evolution of strategies. Economists, too, have analyzed bounded rationality versions of game-theoretic situations, with a particularly advanced formal treatment found in Young (1998).

Current areas of development

To capture recent trends in mathematical sociology, an examination of issues of the premier journal in the field is useful, in particular issues of *The Journal of Mathematical Sociology* (*JMS*) from 2008 to 2011. Several trends stand out: the importance of small group experiments in providing data regularities for formal explanation, the continuing interest in balance as a master concept, the permeability of the boundary between pure mathematical sociology and problems in quantitative methodology, the use of simulations to derive results from models, the perennial interest in the micro–macro theme and the problem of emergence, and the fertility of problems growing out of research on networks of social relations.

Before these trends are inspected in detail, however, it should be pointed out that mainstream sociological journals, like *American Journal of Sociology* and *American Sociological Review*, have been and continue to be hospitable outlets for mathematical sociology articles. Early on these journals published the influential works of Granovetter (1978), who analyzed the emergence of collective action using threshold models and Sørenson (1977), who employed a vacancy chain model as one aspect of a theoretical analysis of status attainment. These journals also published articles that advanced the foundational work of Rapoport on biased net models, namely Skvoretz (1983) and Fararo and Skvoretz (1987). There are many recent examples of mathematical sociology, too numerous to mention, appearing in these journals. This section's focus on the *Journal of Mathematical Sociology* is meant to give the reader a more comprehensive survey of the diversity of work in the field than could be gained from a survey of mathematically oriented work in mainstream sociology journals. However, that such work has appeared and continues to appear in these outlets offers evidence that mathematical sociology has a recognized place at the table of contemporary sociology.

The first trend, the use of data from experiments in small groups to drive formal modeling, is exemplified by Buskens and Van de Rijt (2008), Dogan and Van Assen (2009), Willer and Emanuelson (2008), Gächter and Thoni (2011) and Aksoy and Weesie (2009). The first three use data from experiments on networks in which exchanges can be made between connected actors and the aim to model the observed regularities in differential earnings by positions, on average and over time. The other two articles explore choice behavior in game-like environments designed to replicate public goods and asymmetric investment situations.

A second recent trend in contributions appearing in *JMS* is the idea of balance continuing to motivate research. Deng and Abell (2010) and Abell and Ludwig (2009) use analysis and simulation to investigate how a local rule for sign change toward balance affects the long-term structure of the network of positive and negative ties. Van de Rijt (2011) investigates networks in 'jammed' states in which many triads are unbalanced in the traditional sense yet no change to the valence of any one relation can produce a net reduction in the number of imbalanced triads. Montgomery (2009) investigates balance phenomena when an additional consideration, the awareness of the evaluations of the others, is allowed to vary. Finally, Mrvar and Doreian (2009) reconceptualize the original set of balance ideas as, in network terms, a two mode phenomenon and they explore how blockmodeling techniques from network analysis can be used to characterize balanced structures as two mode blockmodels.

Several articles illustrate a third trend, namely the permeability of the boundary between pure mathematical sociology and problems in quantitative methodology, a field that concentrates primarily on statistical models for data analysis. Snijders (2010), Kejzar et al. (2008) and Ziberna (2008) advance quantitative methods in network analysis. Three other articles address issues in general structural equation modeling and demography: the identification problem in structural equation modeling (Bollen and Bauldry, 2010), modeling time series data (Singer, 2010), demographic mobility indices (Bevaud, 2008) and the two sex problem (Micó et al., 2008).

Simulation studies, the fourth trend, abound. Two of the previously mentioned articles use simulation to illustrate results (Abell and Ludwig, 2009; Deng and Abell, 2010). Fossett (2011) continues the research program of Schelling in using agent-based models to study residential segregation. Flache and Macy (2011) explore opinion polarization on different network typologies with agent-based methods. Helbing et al. (2011) examine the evolution of cooperation in the prisoner's dilemma game in a spatial and a network environment. Fioretti (2010) uses simulation to derive consequences of a stochastic process model for a vacancy chain representation of resource flows. Hevenstone (2009) studies the use of labor market intermediaries (temp agencies) with an agent-based model. Finally, Centola (2009) uses simulation methods to compare complex and simple contagion in different network topologies, ones with scale-free vs exponential degree distributions, subject to disruption by random removal of vertices.

Articles using agent-based models illustrate the fifth trend, interest in the micro–macro problem and emergent phenomena, because they are often set up to demonstrate how micro specifications of behavior can, when agents interact, aggregate to emergent phenomena like waves of cooperation. In addition, however, several articles provide analytical results for such problems. Raub et al. (2011) and Opp (2011) discuss meta-theoretical issues with respect to the micro–macro problem as framed by the 'Coleman boat' image (Coleman, 1990). This image visualizes a macro–macro proposition as explained by three linked processes: a propositional link from macro to micro, a horizontal link from micro to micro (the conceptual location for individual rational choices) and an up-link from micro to macro. The logic of Coleman's boat provides an explanatory frame for a comparative statics macro-level proposition corresponding to an empirical generalization through a dynamic system model at the micro level based on rational choice theory and action.

Other papers also illustrate this trend. Yamaguchi (2011) assesses how population heterogeneity with respect to cost-benefit comparisons affects collective outcomes in two examples, crime and enforcement and gender role attitudes. Jasso (2010) offers a framework for studying the links between micro and macro phenomena that proceeds from a simple basis of populations characterized by variables. Menicucci and Sacco (2009) and Bischi and Merlone (2009) derive limits on the global behavior of agent-based models for prosocial behavior (preferences that are sensitive to the payoffs of others) and for binary choice games with externalities. Finally, Kitts (2008) also derives bounds for collective outcomes for a previously investigated agent-based model of formal and informal control.

Finally, *JMS* continues to attract mathematical work relating to social network analysis. Grassi et al. (2010) prove some general results about the relationship between degree and eigenvector centrality for trees as a class of networks. Friedkin (2010) uses multi-level event history analysis on a classic network data set, the medical innovation data of Coleman et al. (1966), to demonstrate the importance of network position in adoption of tetracycline. Agneessens and Roose (2008) show how exponential random graph models can be used to analyze a two-mode network of persons (theater goers) connected to events (theater performances).

The remaining papers published in these four volumes are an eclectic collection. Three have status as a theme: an evolutionary psychology model linking occupational status and fertility behavior (Hopcroft and Whitmeyer, 2010); inequity effects on behavior in some standard game theory designs (Tutic and Liebe, 2009); and the contingent emergence of a 'Matthew effect' in status systems (Bothner et al., 2010). Three papers look at properties of some special applications of stochastic models, diffusion of rumors (Molchanov and Whitmeyer, 2010), turnover in law firms (Denrell and Shapira, 2009) and success in air combat (Simkin and Roychowdhury, 2008). Two papers by Wyburn and Hayward (2008, 2010) apply standard equilibrium analysis to a system of differential equations modeling the interaction of linguistic majorities and minorities. The remaining three papers fail to fall neatly into any of the above clusters. Pólos et al. (2010) use modal logic to analyze the concept of legitimation; Hajeeh and Lairi (2009) apply an engineering decision analysis method to female selection of marriage partners in Kuwait; and Mayer (2008) applies differential equations to study transformations and deformations in the space of political positions in the USA over time.

This brief and selective survey of recent work in mathematical sociology gives testimony to the vitality and diversity of the field. Topics from the earliest days, like balance and network models, continue to be of contemporary interest. The tools of researchers remain many of the standard and well-known methods of mathematics: differential equations, stochastic processes and game theory. Newer tools like agent-based models are prominently represented. Perennial substantive problems still drive research: diffusion, social influence, status origins and consequences, segregation, cooperation, collective action, power.

Challenges and prospects

Just as perennial as the substantive problems that engage mathematical sociologists are the challenges they face. Mathematical sociologists are or should think of themselves as first and foremost theorists. Putting sociological theory into formal terms is the main task. This is challenging in a field not known for its hospitality to formal theory. There are signs of change: the recent prominence of 'analytical sociology' as a subfield of theoretical sociology is encouraging. Analytical sociologists are very much interested in the mechanisms by which social and collective phenomena are produced: 'analytical sociology explains by detailing the mechanisms through which social facts are brought about and these mechanisms invariably refer to individuals' actions and the relations that link actors to one another' (Hedström and Bearman, 2009: 4). Much of mathematical sociology works to lay bare such mechanisms in ways that make clear how the social facts at issue follow from actions and relations.

A second perennial challenge is relevance. A mathematical model must abstract from everyday experience – it must represent social life in limited terms. Natural scientists, especially physicists, have taken this step with their subject matter – the color of the apple or its taste is irrelevant to its behavior in a gravitational field. No one would think to criticize Newton for irrelevance because all apples have some taste and therefore an apple's taste must be included in any explanation of its flight. In comparison, sociologists often abstract away from the personalities of actors when analyzing a social system. Homans in the *Human Group* specifically sets aside considerations of personality and Simon's model makes this quite clear. Yet there are those who would object to such an abstraction and argue that personality differences must be included in social system accounts because they are there. This overlooks the analytical character of scientific theorizing. However, things are clearly not so settled in sociology. Hence, the adequacy of a representation/abstraction for a particular set of questions can be a contentious issue and even if judged adequate, the importance of the questions may then be challenged. Nevertheless, mathematical sociologists must persevere, believing that, as Doreian (1970: 153) has expressed, 'the future of sociology as a viable discipline will largely depend on the use of mathematics in an informed and imaginative manner'.

Despite these challenges, the long run prospects for work in mathematical sociology are robust. The problems that attract social scientists and the social problems that assail contemporary societies are both problems that at their core have massive interdependency as their signature element. This interdependency among the actions of agents and various levels of social organization cannot be wished away nor can it be assumed irrelevant to a system's trajectory and outcome. Coming to terms with such problems can only be accomplished through formalization and creative use of mathematical modeling and simulation. Heckathorn (2002) shows that important applied results can emerge out of basic research that includes the construction of mathematical models. He describes a theoretical research program that concentrated on such basic sociological problems as how collective action is generated and how social norms emerge but then led to applied research dealing with interventions to prevent the spread of AIDS. In turn, the results of this applied research were implemented in real-life situations. One of his most important conclusions is that 'the stark separation traditionally existing in sociology between theoretic and applied work need not exist' (Heckathorn, 2002: 105). He might have added that the use of mathematical models is a key feature in the success of such efforts and that such success can help to advance the place of mathematical sociology within the broader discipline of sociology.

Annotated further reading

Edling C (2002) Mathematics in sociology. *Annual Review of Sociology* 28: 197–220.
Edling's review of trends in mathematical sociology highlights the convergence among process, structure and action in models of social phenomena. He acknowledges that the rubrics of process, structure and action are common terms to characterize mathematical work but argues that recent trends suggest a blurring of boundaries. Of special interest is the fact that Edling conducted short interviews with leading mathematical sociologists (Peter Abell, Phillip Bonacich, Kathleen Carley, Patrick Doreian, Thomas Fararo and Harrison White) and these interviews inform Edling's observations on the past and future of mathematical sociology.

Freeman LC (1984) Turning a profit from mathematics – the case of social networks. *Journal of Mathematical Sociology* 10: 343–360.
Freeman discusses the early history of mathematics in sociology, both successes and conspicuous failures (the failures deriving from a too literal imitation of the use of mathematics in the physical sciences). He then describes the state of the art (at the time) in social network analysis in which the alliance between mathematics and social science had been extremely productive. For Freeman, such work is 'appropriately mathematical', problem driven rather than purely emulative of what worked in other sciences.

Heise DR (2000) Thinking sociologically with mathematics. *Sociological Theory* 18: 498–504.
Heise recounts his development of affect control theory (ACT) from the seed of a sociological question of how people learn to perform appropriate role actions and avoid deviant behaviors without learning an extensive catalog of dos and don'ts to a full fledged theory with extensive formulation in some very standard mathematics. He points out specific examples of the advantages formalization brought to the sociological intuitions.

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résumé Dans cet article, il décrit l'histoire, les présuppositions, les développements en cours, les épreuves y les perspectives de la sociologie mathématique. Les tâches de recherche courantes dans cet domaine depuis la dernière guerre jusqu'au présent sont identifiées. Il donne des arguments en faveur de l'importance de la sociologie mathématique pour la future de la théorie sociologique.

mots-clés construction des modèles ◆ procédé ◆ sociologie mathématique ◆ structure ◆ théorie sociologique

resumen En este artículo, se describe la historia, las presuposiciones, el desarrollo en curso, los desafíos y las perspectivas para la sociología matemática. Los hilos de investigación comunes en este campo después de la segunda guerra mundial hasta el presente son identificados. Se dan argumentos a favor de la importancia de la sociología matemática para el futuro de la teoría sociológica.

palabras clave construcción de modelos ◆ estructura ◆ proceso ◆ sociología matemática ◆ teoría sociológica